

Revisiting Quantum discord for two-qubit X states: Error bound to Analytical formula

Min Namkung,^{*} Jinho Chang, Jaehee Shin, and Younghun Kwon[†]

Department of Physics, Hanyang University, Ansan, Kyunggi-Do, 425-791, South Korea

In this article, we investigate the error bound of quantum discord, obtained by the analytic formula of Ali et al.[Phys. Rev. A 81(2010), 042105] in case of general X states and by the analytic formula of Fanchini et al.[Phys. Rev. A 81(2010), 052107] in case of symmetric X states. We show that results of Ali et. al. to general X states and Fanchini et al. to symmetric X states may have worst-case error of 0.004565 and 0.0009 respectively.

I. INTRODUCTION

A key ingredient in understanding quantum information may be quantum correlation. A well-known example of quantum correlation is entanglement. An entanglement cannot be obtained by a local operation and classical communication(LOCC)[1, 2]. An entanglement is known to be very fragile to a local noisy channel. Furthermore, it was shown that a quantum state without entanglement contains non-locality[3]. L. Henderson and V. Vedral[4] suggested a method to obtain a classical correlation between parties. H.Ollivier and W.H.Zurek[5] defined a quantum correlation called quantum discord. The quantum discord can be understood as the quantum correlation which is the total correlation minus the classical correlation in a bipartite quantum state. Quantum discord can contain a value even in a separable state[5]. The quantum discord is invariant under unitary operation[4, 5]. In addition, quantum discord seems to have a relation with noisy teleportation, entanglement distillation and quantum state merging[6]. Some experimental effort to check quantum discord are under progress[7, 8].

Quantum discord depends on the measurement setting. In [5] a projective measurement was used in obtaining quantum discord. The measurement for optimal quantum discord should build the maximum of the classical correlation. It was shown in [9] that 2 element optimal POVM should be projective measurements. Quantum discord for a Bell-diagonal state was analytically obtained[10]. Much research has had a focus on finding the quantum discord of the X state[12–18].

There are two reasons why the quantum discord of the two qubit X states becomes significant. The first reason is that the X state can be obtained through a unitary transformation to a two qubit state. The second one is that the general two qubit state can become a X state under a noisy channel[11]. Ali et. al. [12] tried to find an analytic formula of quantum discord to X state. However, it is known that the result of [12] holds only to special X state[13]. Fanchini et. al. [14] tried to find an

analytic formula of the quantum discord for symmetric X state. Recently, by using von Neumann measurement Y. Huang showed that the result of [12] may be valid with some worst-case error[15].

In this report we show that three element POVM can provide a better quantum discord. Shi et. al[16, 17] showed that there are some quantum states where 3 element POVM should be used for optimal quantum discord. The optimality for 3 element POVM can be found by a triangle formed by the direction vectors. By using 3 element POVM, we numerically obtained the quantum discord to the quantum state considered in [15, 18] and compared it with the result of [15].

II. QUANTUM DISCORD

The total correlation between the classical subsystem A and B can be defined by $I(A : B) = H(p^A) + H(p^B) - H(p^A, p^B)$. Here $H(p^X)$ (X can be A or B) is the Shannon entropy of subsystem X. If the probability distribution of the subsystem becomes $p^X = \{p_1^X, p_2^X, \dots, p_n^X\}$, the Shannon entropy is found to be $H(p^X) = -\sum_{i=1}^n p_i^X \log_2 p_i^X$. $H(p^A, p^B)$ is the joint entropy of the total system composed of subsystem A and B. When the probability distribution of the total system is known as $\{p_{ij}^{AB}\}$ ($i = 1, 2, \dots, n, j = 1, 2, \dots, m$), joint entropy is found to be $H(p^A, p^B) = -\sum_{i,j=1}^{n,m} p_{ij}^{AB} \log_2 p_{ij}^{AB}$.

Let us consider the quantum case. In quantum information one may consider the information of the quantum system as the quantum state corresponding to the system. Let ρ^{AB} denote the quantum state to total system. Then the quantum states of subsystem A and B can be found by $\rho^A = \text{Tr}_B \rho^{AB}$ and $\rho^B = \text{Tr}_A \rho^{AB}$. The Von Neumann entropy of X and the total subsystem are given by $S(\rho^X) = -\text{Tr}\{\rho^X \log_2 \rho^X\}$ (X becomes A or B) and $S(\rho^{AB}) = -\text{Tr}\{\rho^{AB} \log_2 \rho^{AB}\}$ respectively. Therefore the total correlation of the quantum case is expressed by

$$I(A : B) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}). \quad (1)$$

Total correlation given by Eq(1) contains the classical and quantum correlation. Therefore in order to extract the quantum correlation, one has to subtract the classical

^{*}Electronic address: mslab.nk@gmail.com

[†]Electronic address: yyhkwon@hanyang.com

correlation from the total correlation. When one considers the positive operator valued measurement (POVM) $\{M_k^B\}$ on subsystem B , the classical correlation can be defined by Eq.(2)[4]

$$J(A|\{M_k^B\}) = S(\rho^A) - \min_{\{M_k^B\}} \sum_k p_k S(\rho_k^A). \quad (2)$$

Here ρ_k^A is the state of subsystem A , given as $\rho_k^A = \text{Tr}_B\{(1 \otimes M_k^B)\rho^{AB}\}$. $S(A|\{M_k^B\}) = \sum_k p_k S(\rho_k^A)$ is the conditional entropy after measurement of subsystem B . Therefore the quantum correlation between subsystem A and B becomes[5]

$$\begin{aligned} \delta_{\{M_k^B\}}(A : B) &= I(A : B) - J(A|\{M_k^B\}) \\ &= S(\rho^B) - S(\rho^{AB}) + \min_{\{M_k^B\}} \sum_k p_k S(\rho_k^A). \end{aligned} \quad (3)$$

This implies that optimizing Eq.(3) is identical to find a measurement to minimize the conditional entropy $S(A|\{M_k^B\})$. $S(A|\{M_k^B\})$ is unitary invariant[4].

The X state which appears in various physical cases[19, 20] is known to persist under local noisy channel[21]. The Bell diagonal state and the Werner state[22] belong to the X state. The general form of subsystem ρ^{AB} in two qubit states becomes

$$\begin{aligned} \rho^{AB} &= \frac{1}{4} \{ \mathcal{I} \otimes \mathcal{I} + \sum_i (A_i \mathcal{I} \otimes \sigma_i + B_i \sigma_i \otimes \mathcal{I}) \\ &\quad + \sum_{i,j} t_{ij} \sigma_i \otimes \sigma_j \}. \end{aligned} \quad (4)$$

Here $\sigma_i (i = 1, 2, 3)$ is Pauli's spin matrices and $\vec{A} = (A_1, A_2, A_3)$, $\vec{B} = (B_1, B_2, B_3)$ and t_{ij} can be found by

$$\begin{aligned} A_i &= \text{Tr}\{(\mathcal{I} \otimes \sigma_i)\rho^{AB}\}, \\ B_i &= \text{Tr}\{(\sigma_i \otimes \mathcal{I})\rho^{AB}\}, \\ t_{ij} &= \text{Tr}\{\sigma_i \otimes \sigma_j \rho^{AB}\}. \end{aligned} \quad (5)$$

Without loss of generality, one may assume that all the parameters are real. By applying unitary operations to Eq.(4), one can get the X state ρ_X^{AB}

$$\rho_X^{AB} = \begin{pmatrix} a & 0 & 0 & \epsilon \\ 0 & b & \delta & 0 \\ 0 & \delta & c & 0 \\ \epsilon & 0 & 0 & d \end{pmatrix}. \quad (6)$$

Here, every element of ρ_X^{AB} is real. a, b, c and d satisfies $a + b + c + d = 1$. Eq. (6) can be expressed as

$$\begin{aligned} \rho_X^{AB} &= \frac{1}{4} (\mathcal{I} \otimes \mathcal{I} + A\mathcal{I} \otimes \sigma_3 + B\sigma_3 \otimes \mathcal{I} \\ &\quad + \sum_i t_i \sigma_i \otimes \sigma_i). \end{aligned} \quad (7)$$

Here A, B, t_1, t_2 and t_3 in Eq. (7) become

$$\begin{aligned} A &= a - b + c - d, \\ B &= a + b - c - d, \\ t_1 &= 2(\delta + \epsilon), \\ t_2 &= 2(\delta - \epsilon), \\ t_3 &= a - b - c + d. \end{aligned} \quad (8)$$

III. OPTIMIZATION STRATEGY

One may ask whether there exists an optimal POVM for conditional entropy compared to projective measurement. In [9] it was shown that projective measurement is the optimal condition for the 2 element POVM. Therefore, one should consider more than two elements POVM. In [17], they found that there is X state where 3 element POVM can be optimal. As is well known, it is very difficult to handle the optimal 3 element POVM analytically. The general form of 3 element POVM is expressed as [13]

$$M_k^B = \mu_k (\mathcal{I} + \vec{n}^{(k)} \cdot \vec{\sigma}), \quad k = 1, 2, 3, \quad \mu_k > 0. \quad (9)$$

Here, $\vec{n}^{(k)}$ is the direction vector to M_k^B . Since, $|\vec{n}^{(k)}| = 1$, the positivity of M_k^B holds. When subsystem B is measured by $\{M_k^B\}$, the post-measurement state ρ_k^A of subsystem A becomes $\rho_k^A = [1 + \{t_1 m_x^{(k)} \sigma_1 + t_2 m_y^{(k)} \sigma_2 + (t_3 m_z^{(k)} + B) \sigma_3\} / (1 + A m_z^{(k)})] / 2$, where $m_i^{(k)} (i = x, y, z)$ is a component of the k th element in the direction vector $\vec{m}^{(k)}$. The eigenvalues of ρ_k^A are $\{1 \pm E(m_x^{(k)}, m_y^{(k)}, m_z^{(k)})\} / 2$. Here $E(m_x^{(k)}, m_y^{(k)}, m_z^{(k)})$ is defined by

$$\begin{aligned} E(m_x^{(k)}, m_y^{(k)}, m_z^{(k)}) &= \frac{\sqrt{(t_1 m_x^{(k)})^2 + (t_2 m_y^{(k)})^2 + (t_3 m_z^{(k)} + B)^2}}{1 + A m_z^{(k)}}. \end{aligned} \quad (10)$$

The probability to obtain outcome k turns out to be $p_k = \mu_k (1 + m_z^{(k)} A)$. Therefore when 3 element POVM is used for measurement, the conditional entropy $S(A|\{M_k^B\})$ can be found as

$$\begin{aligned} S(A|\{M_k^B\}) &= \sum_{k=1}^3 \mu_k (1 + A m_z^{(k)}) h(E(m_x^{(k)}, m_y^{(k)}, m_z^{(k)})). \end{aligned} \quad (11)$$

Here $h(x)$ is a function defined as $h(x) = -\frac{1+x}{2} \log_2 \frac{1+x}{2} - \frac{1-x}{2} \log_2 \frac{1-x}{2}$. The complete condition to $\{M_k^B\}$ becomes [13]

$$\mu_1 + \mu_2 + \mu_3 = 1, \quad (12)$$

$$\mu_1 \vec{n}^{(1)} + \mu_2 \vec{n}^{(2)} + \mu_3 \vec{n}^{(3)} = 0. \quad (13)$$

When there are three POVM elements, the direction vector for each element forms a triangle. The shape of this

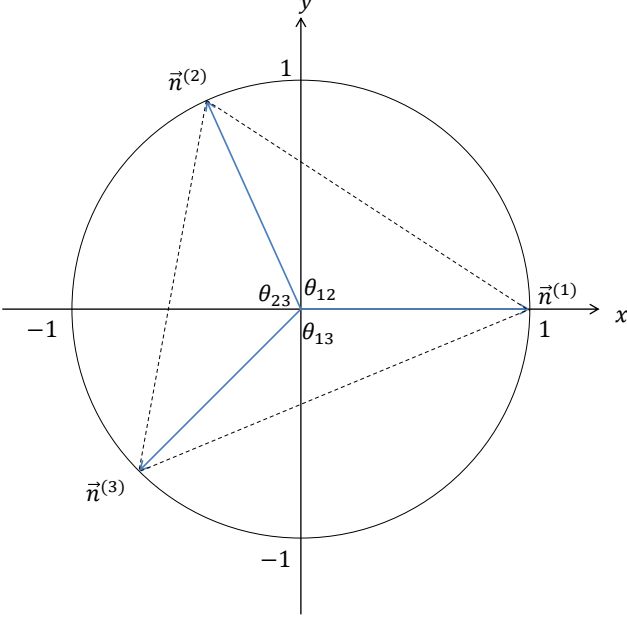


FIG. 1: A triangle composed of the direction vector $\vec{n}^{(1)}, \vec{n}^{(2)}, \vec{n}^{(3)}$

triangle depends on μ_1, μ_2 and μ_3 . Fig. 1 shows a triangle made by $\vec{n}^{(1)}, \vec{n}^{(2)}$ and $\vec{n}^{(3)}$ in the XY plane. θ_{ij} denotes the angle between the direction vectors $\vec{n}^{(i)}$ and $\vec{n}^{(j)}$. From Eq.(12)-(13) one can obtain three equations to those angles

$$\begin{aligned} \mu_1 + \mu_2 \cos \theta_{12} + \mu_3 \cos \theta_{13} &= 0, \\ \mu_1 \cos \theta_{12} + \mu_2 + \mu_3 \cos \theta_{23} &= 0, \\ \mu_1 \cos \theta_{13} + \mu_2 \cos \theta_{23} + \mu_3 &= 0. \end{aligned} \quad (14)$$

From Eq.(14) the relations between $\theta_{12}, \theta_{23}, \theta_{13}$ and μ_1, μ_2, μ_3 can be given as

$$\begin{aligned} \theta_{12} &= \cos^{-1} \frac{\mu_3^2 - \mu_1^2 - \mu_2^2}{2\mu_1\mu_2}, \\ \theta_{23} &= \cos^{-1} \frac{\mu_1^2 - \mu_2^2 - \mu_3^2}{2\mu_2\mu_3}, \\ \theta_{13} &= \cos^{-1} \frac{\mu_2^2 - \mu_1^2 - \mu_3^2}{2\mu_1\mu_3}. \end{aligned} \quad (15)$$

The condition where θ_{12}, θ_{23} , and θ_{13} are real can be found from $-1 < \cos \theta_{12}, \cos \theta_{23}, \cos \theta_{13} < 1$, which becomes Eq.(16). Fig. 2 displays the region for (μ_1, μ_2) where θ_{12}, θ_{23} , and θ_{13} are real.

$$\begin{aligned} |\mu_2 - \mu_3| &< \mu_1 < \mu_2 + \mu_3 \\ |\mu_1 - \mu_3| &< \mu_2 < \mu_1 + \mu_3 \\ |\mu_1 - \mu_2| &< \mu_3 < \mu_1 + \mu_2 \end{aligned} \quad (16)$$

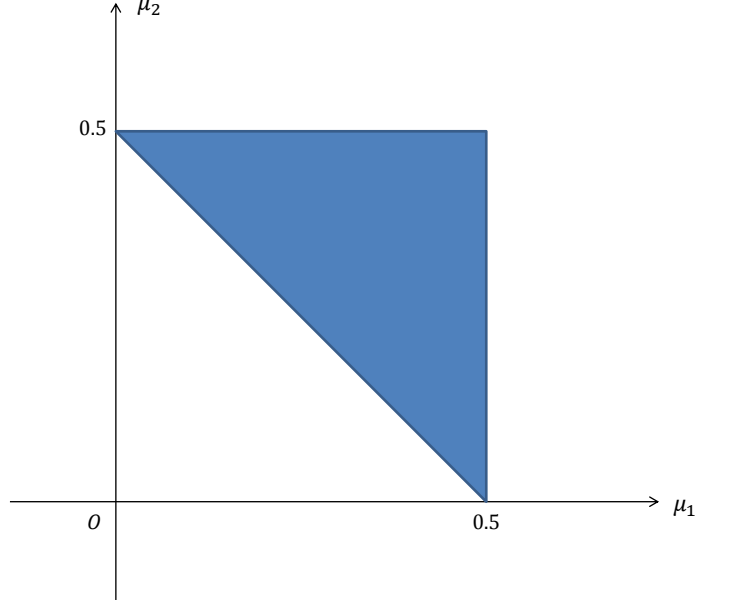


FIG. 2: The permitted region of (μ_1, μ_2) for 3 element POVM. The edges are excluded.

However, the measurement illustrated in Fig. 1 does not describe the most general POVM. In order to indicate the most general POVM one has to consider not the XY plane but an arbitrary plane. Therefore to find the direction vectors in an arbitrary plane, one can rotate them in Fig. 1 using the Euler angle. The completeness holds under the rotation of the direction vectors. There are three rotation matrices in Eq.(17)

$$R(\psi, \theta, \phi) = R_\psi R_\theta R_\phi, \quad (17)$$

Where each rotation matrices is

$$\begin{aligned} R_\psi &= \begin{pmatrix} \cos \psi & 0 & \sin \psi \\ 0 & 1 & 0 \\ -\sin \psi & 0 & \cos \psi \end{pmatrix}, \\ R_\theta &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}, \\ R_\phi &= \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \quad (18)$$

The direction vector of Fig. 1 are $\vec{n}^{(1)} = (1, 0, 0)$, $\vec{n}^{(2)} = (\cos \theta_{12}, \sin \theta_{12}, 0)$, $\vec{n}^{(3)} = (\cos \theta_{13}, -\sin \theta_{13}, 0)$. Through rotation matrix $R(\psi, \theta, \phi)$, one can obtain new direction

vectors such as

$$\begin{aligned}
\vec{m}^{(1)} &= R(\psi, \theta, \phi) \vec{n}^{(1)} \\
&= \begin{pmatrix} \cos \phi \cos \psi + \sin \phi \sin \psi \sin \theta \\ \cos \theta \sin \phi \\ \sin \phi \cos \psi \sin \theta - \cos \phi \sin \psi \end{pmatrix}, \\
\vec{m}^{(2)} &= R(\psi, \theta, \phi) \vec{n}^{(2)} \\
&= \begin{pmatrix} \cos(\theta_{12} + \phi) \cos \psi + \sin(\theta_{12} + \phi) \sin \psi \sin \theta \\ \sin(\theta_{12} + \phi) \cos \theta \\ -\cos(\theta_{12} + \phi) \sin \psi + \sin(\theta_{12} + \phi) \cos \psi \sin \theta \end{pmatrix}, \\
\vec{m}^{(3)} &= R(\psi, \theta, \phi) \vec{n}^{(3)} \\
&= \begin{pmatrix} \cos(\theta_{13} - \phi) \cos \psi - \sin(\theta_{13} - \phi) \sin \psi \sin \theta \\ -\sin(\theta_{13} - \phi) \cos \theta \\ -\cos(\theta_{13} - \phi) \sin \psi - \sin(\theta_{13} - \phi) \cos \psi \sin \theta \end{pmatrix}.
\end{aligned} \tag{19}$$

It is difficult to optimize Eq.(11) analytically. Therefore we use a Monte-Carlo simulation for optimizing Eq.(11). Our strategy is as follows. We randomly select (μ_1, μ_2, μ_3) in the region of Fig. 2. We examine a minimum conditional entropy in the region $[0, 2\pi]$ to the Euler angle ψ, θ, ϕ . It is found that a minimum conditional entropy does not depend on ϕ .

Y. Huang and Lu et. al considered quantum discord of special X states such as

$$\rho_1^{AB} = \begin{pmatrix} 0.027180 & 0 & 0 & 0.141651 \\ 0 & 0.000224 & 0 & 0 \\ 0 & 0 & 0.027327 & 0 \\ 0.141651 & 0 & 0 & 0.945269 \end{pmatrix} \tag{20}$$

$$\rho_2^{AB} = \begin{pmatrix} 0.021726 & 0 & 0 & 0.128057 \\ 0 & 0.010288 & 0 & 0 \\ 0 & 0 & 0.010288 & 0 \\ 0.128057 & 0 & 0 & 0.957698 \end{pmatrix} \tag{21}$$

and

$$\rho_3^{AB} = \begin{pmatrix} 0.0783 & 0 & 0 & 0 \\ 0 & 0.1250 & 0.1000 & 0 \\ 0 & 0.1000 & 0.1250 & 0 \\ 0 & 0 & 0 & 0.6717 \end{pmatrix}. \tag{22}$$

In ref.[15] they treated the quantum discord of the X state using projective measurement. One can see that the condition to the maximum of discord turns out to be $\theta = \pi/2$. Quantum discords for the X state of Eq.(20), Eq.(21)[15] and Eq.(22)[18] does not change dramatically according to the measurement setting.

Table I shows the quantum discord of 3 element POVM, that of 2 projective measurements and that of measurement obtained by Ali et al. (δ_2) respectively. As we can see, $\delta_{3,\min}$ for ρ_1^{AB} becomes 0.123010 which is 0.001613 less than the minimum value of the quantum discord obtained from 2 projective measurement in [15] (The quantum discord obtained here is a little different from the result of [15]). It is because the quantum discord obtained in ref.[15] were expressed in terms

X state	$\delta_{3,\min}$	$\delta_{2,\min}$	δ_2
ρ_1^{AB}	0.123010	0.124623	0.127575
ρ_2^{AB}	0.107873	0.107948	0.108773
ρ_3^{AB}	0.132730	0.132741	0.132751

TABLE I: Quantum discord for the states shown in Eq.(20)-(22) when 3 element POVM($\delta_{3,\min}$), projective measurement($\delta_{2,\min}$) and measurement obtained by Ali et al. (δ_2) are used respectively. (μ_1, μ_2, μ_3) which minimizes the quantum discord can be found at (0.4209, 0.2938, 0.2853) for ρ_1^{AB} , (0.4663, 0.2489, 0.2848) for ρ_2^{AB} and (0.2748, 0.2853, 0.4349) for ρ_3^{AB} respectively.

X state	$\Delta_3(\delta_{3,\min} - \delta_2)$	$\Delta_2(\delta_{2,\min} - \delta_2)$
ρ_1^{AB}	-0.004565	-0.002952
ρ_2^{AB}	-9.0030×10^{-4}	-8.2542×10^{-4}
ρ_3^{AB}	-2.1109×10^{-5}	-9.6477×10^{-6}

TABLE II: Difference between the quantum discord of 3 element POVM (2 projective measurement) and that of Ali et al., which is denoted by $\Delta_3(\Delta_2)$.

of the natural logarithm(\log_e). In this paper Every results to quantum discord are obtained in terms of \log_2 . The results to quantum discord in terms of the natural logarithm(\log_e) can be found in Appendix.) In addition, $\delta_{3,\min}$ for ρ_2^{AB} becomes 0.107873 which is 0.000075 less than the minimum value of the quantum discord obtained from 2 projective measurement in [15]. Furthermore $\delta_{3,\min}$ for ρ_3^{AB} becomes 0.132730 which is 0.000011 less than the minimum value of the quantum discord obtained from 2 projective measurement. For 3 element POVM, the optimized values to (μ_1, μ_2, μ_3) turn out to be (0.4209, 0.2938, 0.2853) for ρ_1^{AB} , (0.4663, 0.2489, 0.2848) for ρ_2^{AB} and (0.2748, 0.2853, 0.4349) for ρ_3^{AB} respectively. Furthermore we can see that a better bound for ρ_1^{AB} , ρ_2^{AB} and ρ_3^{AB} can be obtained from 3 element POVM. F. Fanchini et. al provided an analytic formula for the symmetric X state[14]; however, it is known that the formula may not be optimal. It is shown that quantum discord to ρ_2^{AB} provides a lower value with 0.000075 than that of F. Fanchini et. al. Table I and II clearly show that for the quantum states ρ_1^{AB} , ρ_2^{AB} and ρ_3^{AB} , 3 element POVM provide a better value to quantum discord.

IV. CONCLUSION

In this article we investigated the quantum discord to X states considered by Y. Huang and Lu et. al. We investigated the worst error to quantum discord from the analytic formula obtained by Ali et al. in case of general X states and by the analytic formula of Fanchini et al. in case of symmetric X states. By using projective measurement Y. Huang found the worst error to the quantum discord obtained by Ali et. al. to be 0.002952. In this paper we extend the worst case error to 0.004565, by using 3 element POVM. Furthermore for symmetric

two-qubit X states, it was found that by using 3 element POVM that the analytical formula derived by F. F. Fanchini et al. is valid with worst-case error of 0.0009. In addition, 3 element POVM was found to supply better quantum discord for the state considered in Lu et. al. We numerically simulated the lower bound to the quantum states considered by Y. Huang and Lu et. al. However we still need to provide an analytic optimal bound for these states, which is in progress.

Acknowledgement

This work is supported by the Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (NRF-2010-0025620).

Appendix. Quantum discord expressed in terms of the natural logarithm(\log_e).

In the appendix we supply results to quantum discord in terms of the natural logarithm(\log_e).

X state	$\delta_{3,\min}$	$\delta_{2,\min}$	δ_2
ρ_1^{AB}	0.085264	0.086381	0.088428
ρ_2^{AB}	0.074772	0.074824	0.075396
ρ_3^{AB}	0.092001	0.092009	0.092016

TABLE III: Revisited quantum discord for the states shown in Eq.(20)-(22) when 3 element POVM($\delta_{3,\min}$), projective measurement($\delta_{2,\min}$) and measurement obtained by Ali et al.(δ_2) are used respectively. The values are expressed in terms of the natural logarithm(\log_e).

X state	$\Delta_3(\delta_{3,\min} - \delta_2)$	$\Delta_2(\delta_{2,\min} - \delta_2)$
ρ_1^{AB}	-0.003164	-0.002046
ρ_2^{AB}	-6.2400×10^{-4}	-5.7214×10^{-4}
ρ_3^{AB}	-1.4631×10^{-5}	-6.6871×10^{-6}

TABLE IV: Difference between the quantum discord of 3 element POVM(2 projective measurement) and that of Ali et al., which is denoted by $\Delta_3(\Delta_2)$. The values are expressed in terms of the natural logarithm(\log_e).

-
- | | |
|--|---|
| <p>[1] M. B. Plenio and S. Virmani, Quantum Inf. Comput. 7(2007), 1</p> <p>[2] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. 81(2009), 865</p> <p>[3] C. H. Bennett, D. P. Divincenzo, C. A. Fuchs, T. Mor, E. Rains, P. W. Shor, J. A. Smolin, and W. K. Wootters, Phys. Rev. A 59(1999), 1070</p> <p>[4] L. Henderson and V. Vedral, J. Phys. A:Math. Gen. 34(2001), 6899</p> <p>[5] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88(2001), 017901</p> <p>[6] V. Madhok, A. Datta, Int. J. of Mod. Phys. B, 27(2013), 1245041</p> <p>[7] B. P. Lanyon, M. Barbieri, M. P. Almeida and A. G. White, Phys. Rev. Lett. 101(2008), 200501</p> <p>[8] M. Gu, H. M. Chrzanowski, S. M. Assad, T. Symul, K. Modi, T. C. Ralph, V. Vedral, and P. K. Lam, Nat. Phys. 8(2012), 671</p> <p>[9] S. Hamieh, R. Kobes, and H. Zaraket, Phys. Rev. A 70(2004), 052325</p> <p>[10] S. Luo, Phys. Rev. A 77(2008), 042303</p> <p>[11] J. Maziero, T. Werlang, F.F. Fanchini, L.C. Celeri, and</p> | <p>R.M. Serra, Phys. Rev. A 81(2010), 022116</p> <p>[12] M. Ali, A.R.P. Rau, and G. Alber, Phys. Rev. A 81(2010), 042105</p> <p>[13] Q. Chen, C. Zhang, S. Yu, X.X. Yi, and C.H. Oh, Phys. Rev. A 84(2011), 042313</p> <p>[14] F.F. Fanchini, T. Werlang, C. A. Brasil, L. G. E. Arruda, and A. O. Caldeira, Phys. Rev. A 81(2010), 052107</p> <p>[15] Y. Huang, Phys. Rev. A 88(2013), 014302</p> <p>[16] M. Shi, W. Yang, F. Jiang, and J. Du, J. Phys. A:Math. Theor. 44(2011), 415304</p> <p>[17] M. Shi, F. Jiang, X. Yan, and J. Du, Phys. Rev. A 85(2012), 064104</p> <p>[18] X.M. Lu, J. Ma, Z. Xi and X. Wang, Phys. Rev. A 83(2011), 012327</p> <p>[19] J. S. Pratt, Phys. Rev. Lett. 93(2004), 237205</p> <p>[20] S. Bose, I. Fuentes-Guridi, P. L. Knight, and V. Vedral, Phys. Rev. Lett. 87(2001), 050401</p> <p>[21] T. Yu and J. H. Eberly, J. Quant. Inform. and Comput. 7(2007), 459</p> <p>[22] R. Werner, Phys. Rev. A 40(1989), 4277</p> |
|--|---|